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Caltech

Operator algebras in AdS/CFT

Bulk reconstruction, quantum extremal surfaces, and baby universes

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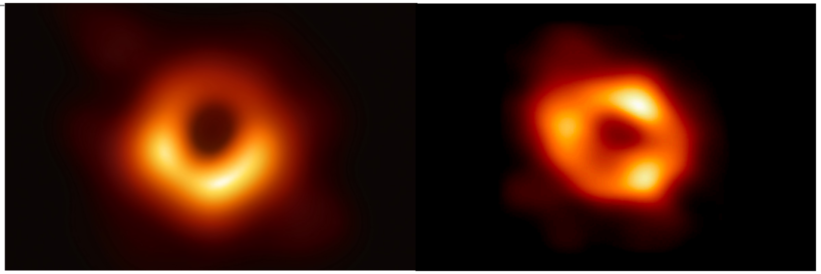
arXiv:1811.05482, 1910.06328 [MJK, David Kolchmeyer]
arXiv:2005.05971, 2005.07189, 2006.14620, 2112.12789 [Elliott Gesteau, MJK]

How to understand Black Holes?

Event Horizon Telescope
2019, 2022



Two prime epitome of modern theoretical physics

General Relativity + Quantum Field Theory

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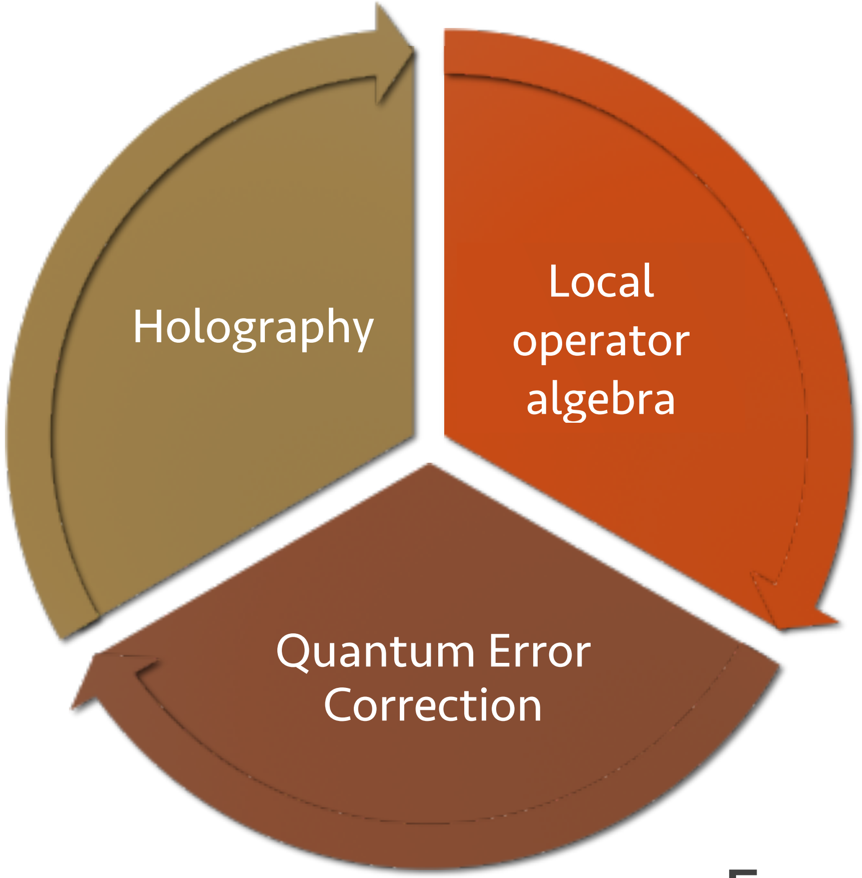
Quantum Gravity

What if I can tell you we can describe it with operator algebras?

Three Ingredients

Goal

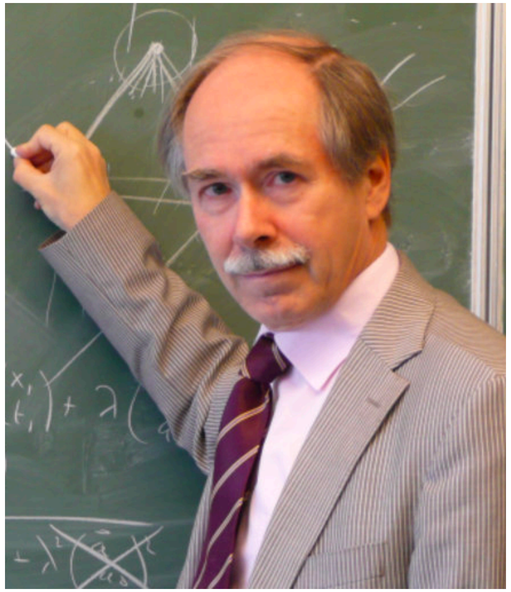


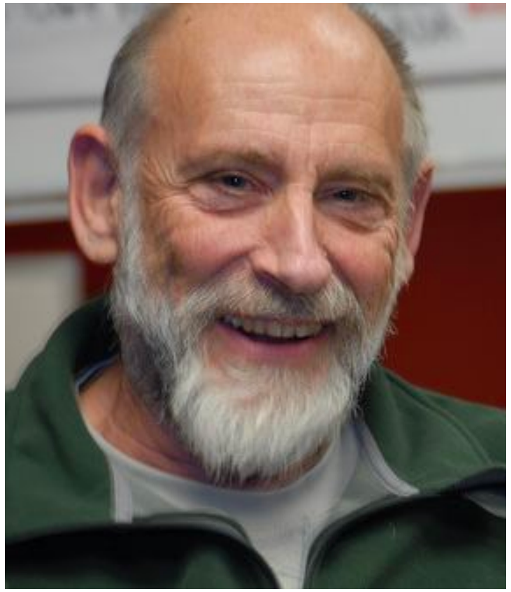
Framework

Emerging structure

The Holographic Principle

'**t Hooft** and **Susskind**





The **information** contained **inside** the BH

↑ *entirely encoded*

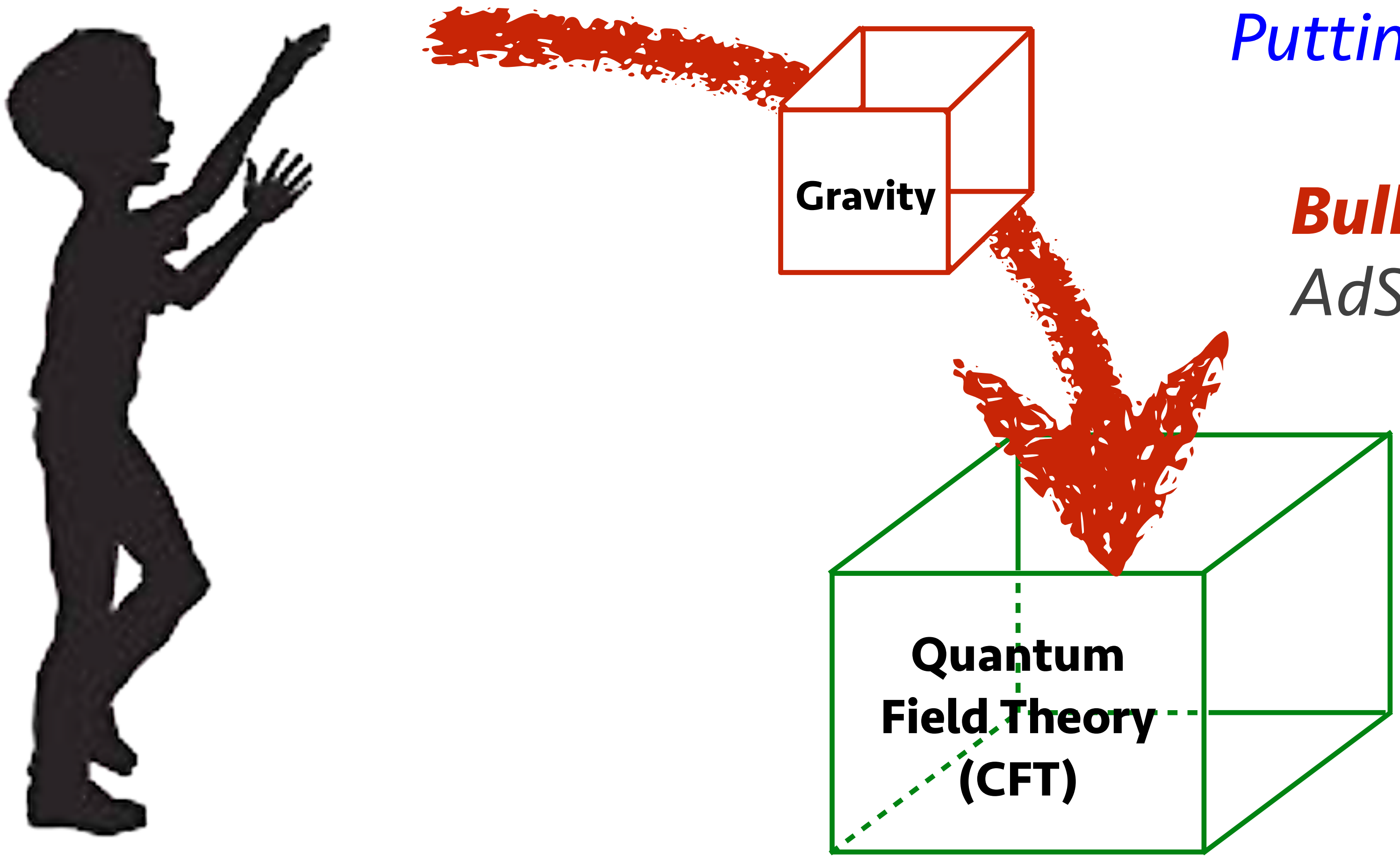
The **fluctuations** of the **boundary** of the BH

(i.e. the horizon)

More generally : any theory describing a **volume** of **spacetime**

⟷ a theory on the **boundary** of that **spacetime**

AdS/CFT

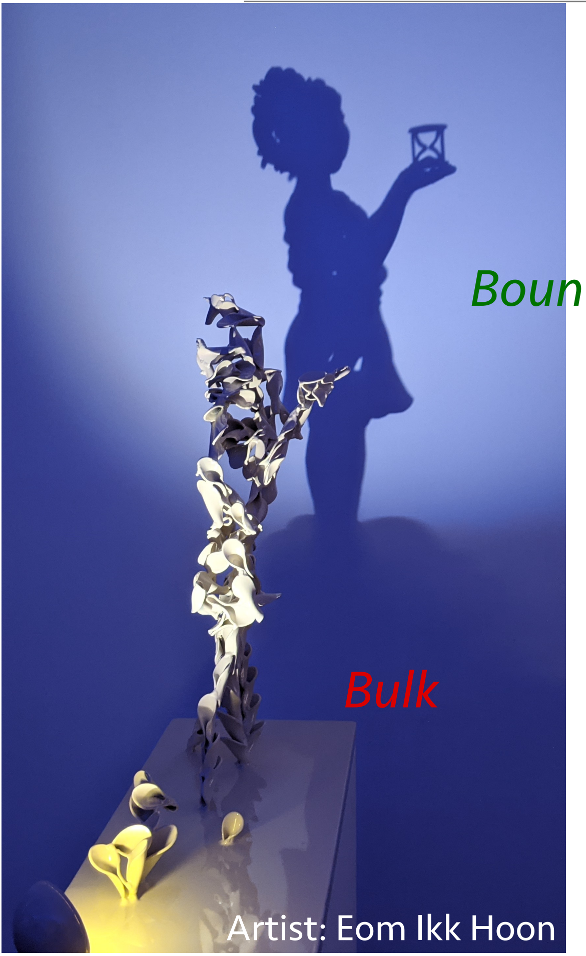


Putting gravity in a box!

Bulk = *Gravity with*
AdS boundary conditions

Boundary =
Non-gravitational
gauge theory

AdS/CFT correspondence [Maldacena]



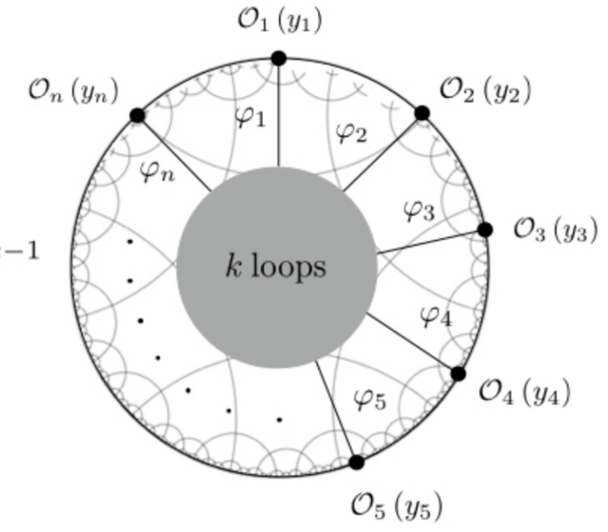
Conformal field theory
in d-dimension

↕

Quantum gravity in
(d+1)-dim AdS space

[Witten]

$$\langle \mathcal{O}_1(y_1) \ldots \mathcal{O}_n(y_n) \rangle_{\text{conn.}} = \sum_{k=0}^{\infty} G^{k-1}$$



Correlation functions can be matched

Hard questions in quantum gravity can be replaced with easier questions in conformal field theory!

***Reverse**: we can use weakly-coupled gravity to study strongly-coupled quantum field theory.*

Operator picture from holography

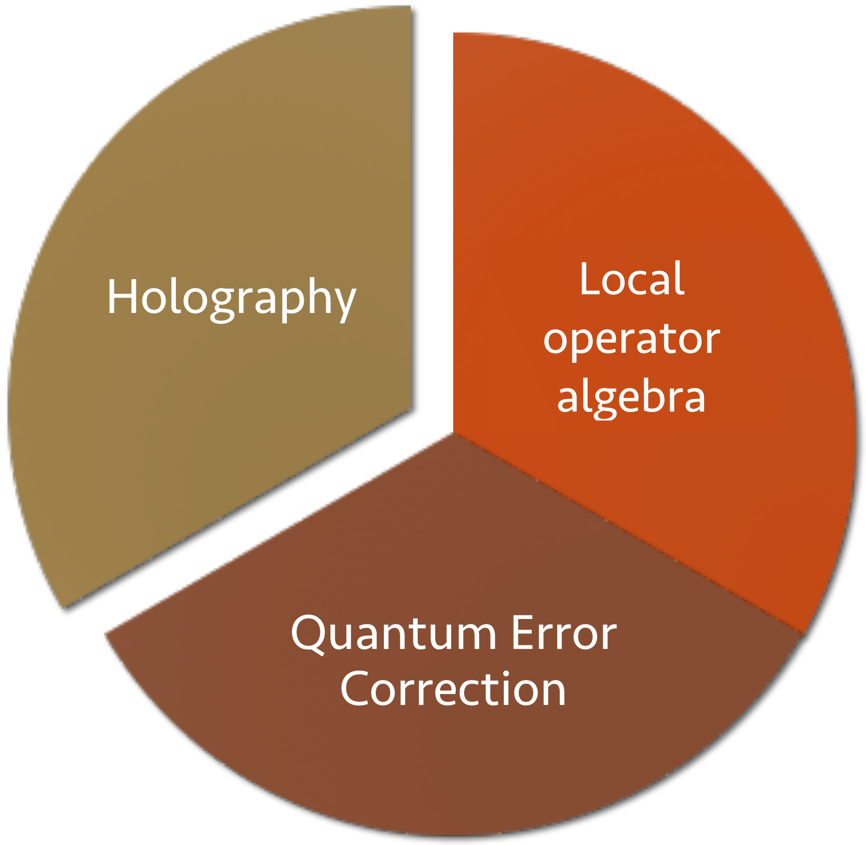
Bulk ⟷ Boundary

Local bulk operators

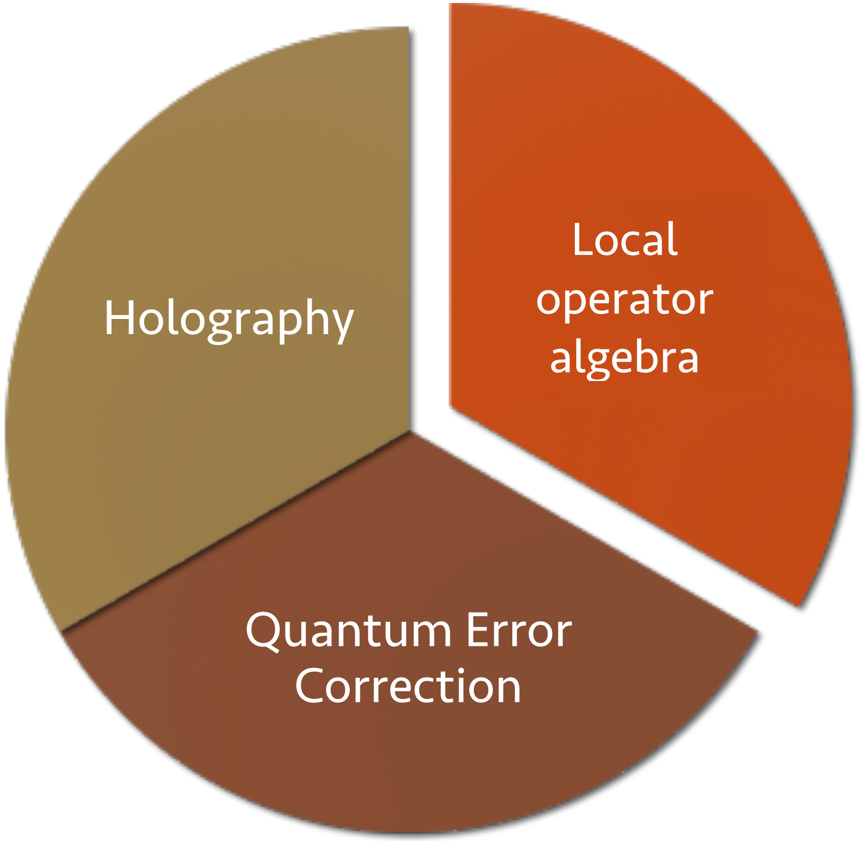
⇓

Boundary operators

smeared over the entire spatial slice or a compact spatial subregion



Local operators form operator algebra



Local operator algebra

$=$ von Neumann algebra

While most QG are done with finite density matrices:

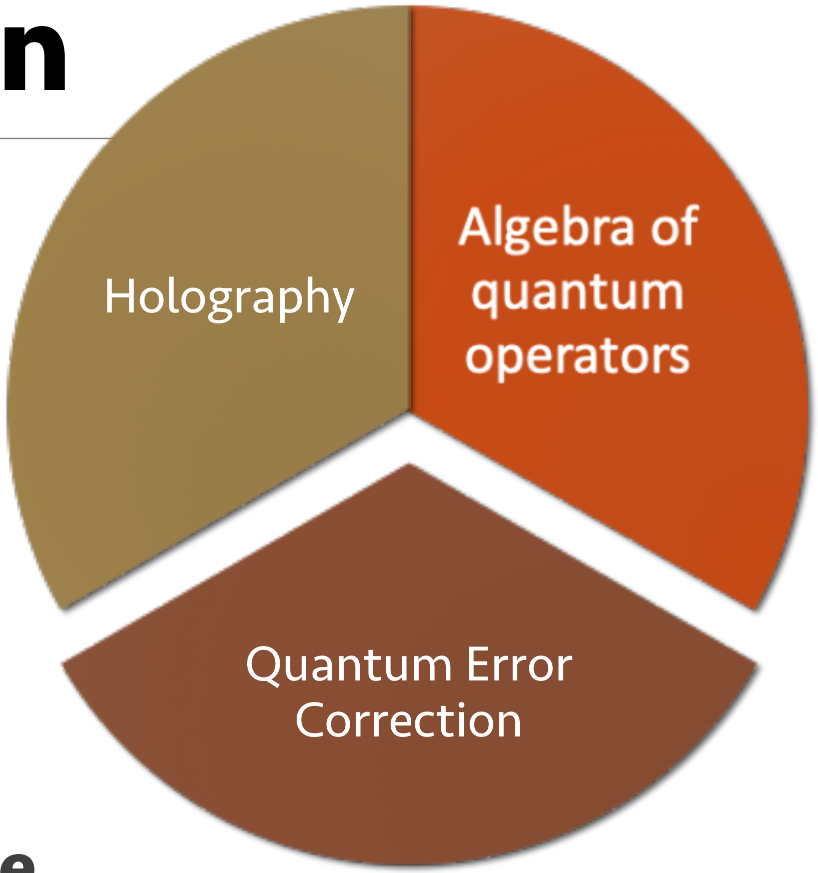
- Finite-dimensional Hilbert space: I_n
- Infinite-dimensional Hilbert space:

$$\mathrm{I}_\infty,\ \mathrm{II}_1,\ \mathrm{II}_\infty,\ \boxed{\mathrm{III}_\lambda}\ (0 \le \lambda \le 1)$$

Type III_1 gives a continuous spectrum needed!

vN algebras used in the context of quantum gravity: [Witten][Harlow][MJK, Kolchmeyer][Gesteau, MJK][Faulkner et al]

Quantum Error Correction



Local bulk operators

utilizing
von Neumann algebra

⇓

Code subspace

of the physical Hilbert space of the boundary CFT

Emerging **quantum error correcting structure**

ex) Quantum Extremal Surface + Bulk Reconstruction ⟷ Complete recovery

vN algebras in QFT: spacetime attached

Unbounded!

Open region of spacetime $u \xrightarrow{\text{Define}} A(u)$ An associated local operator algebra

Polar decomposition

A partial isometry

A self-adjoint positive operator

Canonically associated with a set of projections (Spectral theorem)

Von Neumann algebra $M(u) \subset B(\mathscr{H})$ is generated by

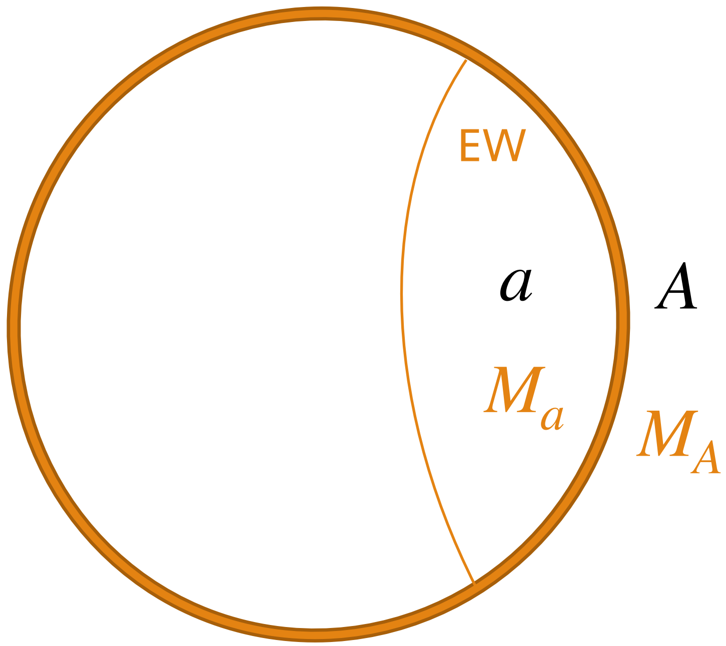
Partial isometries

associated with the operators in M(u)

A set of all projections

$\Rightarrow$ *Denote subregions in the bulk & the boundary*

vN algebra and AdS/CFT geometry



➡ Bulk theory may be effectively described by a QFT on a global AdS background

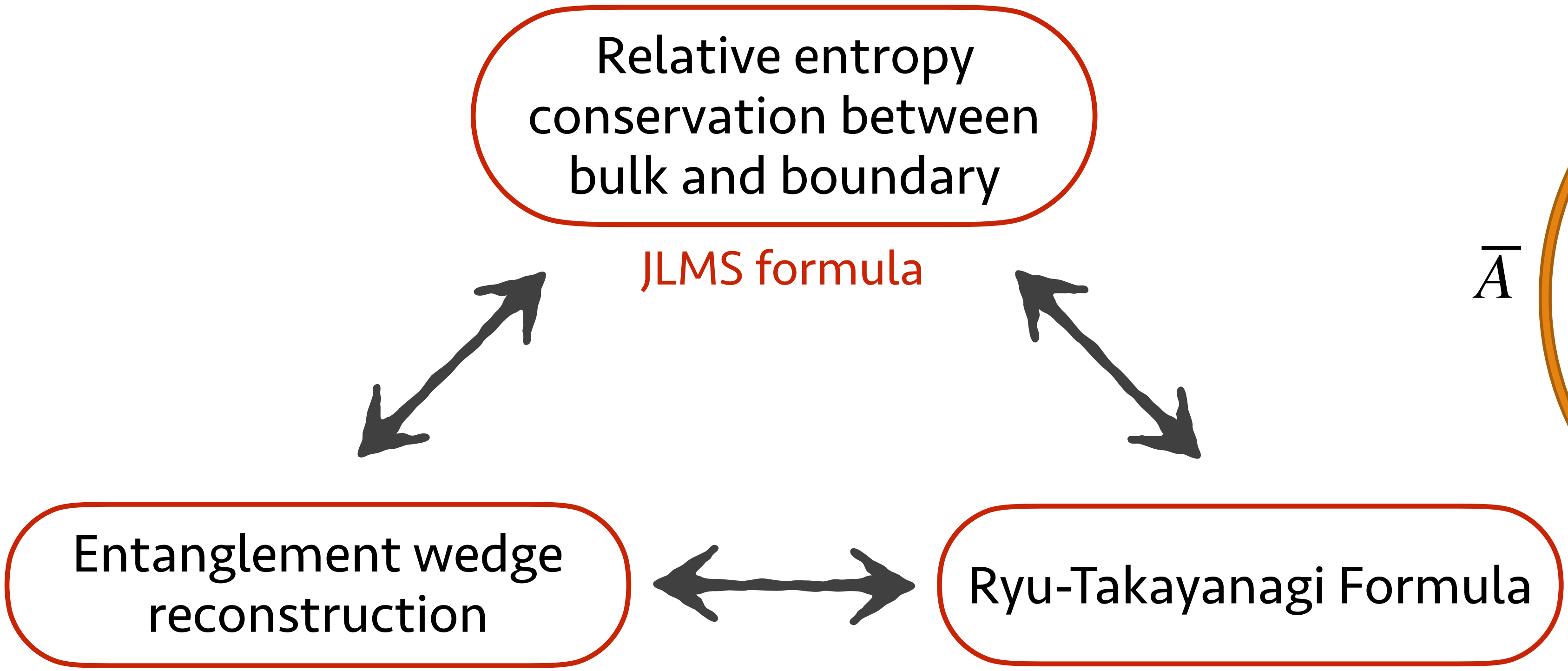
➡ Use von Neumann algebras to describe

operators associated with
covariantly defined subregions in the bulk

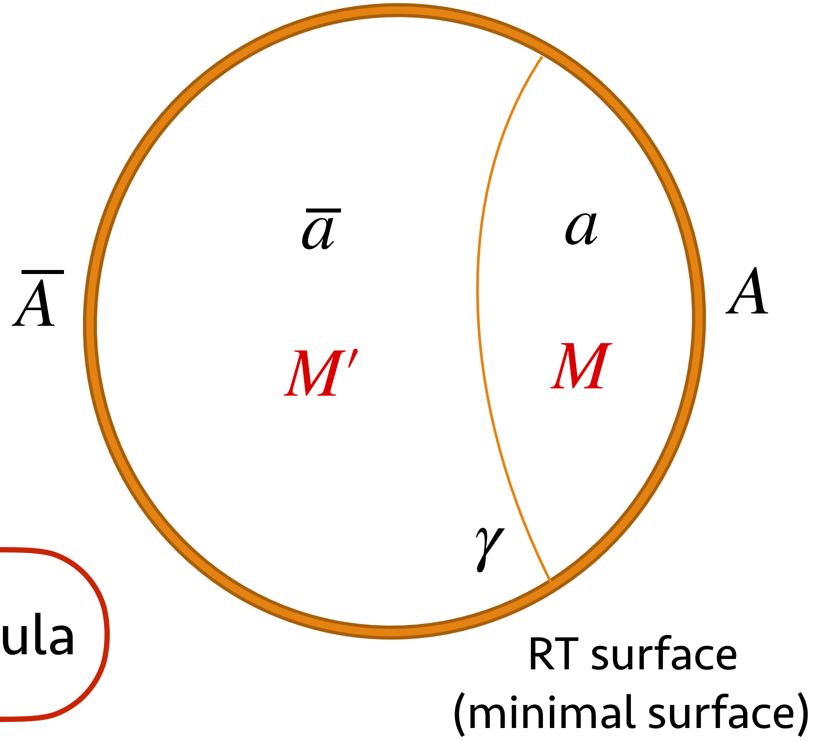
i.e. Entanglement wedge of a boundary subregion

- *Causally complete*
- *Naturally have an associated von Neumann algebra*

Finite-dimensional Hilbert space



[Swingle][Jafferis, Lewkowycz, Maldacena, Suh][Dong, Harlow, Wall]
[Harlow][MJK, Kolchmeyer][Gesteau, MJK][Faulkner]



ρ_{bulk} : the state that describes the system in the semiclassical theory (=the code subspace)

$$S_{gen} := \frac{1}{4G_N}\text{Area}(a) + S_a(\rho_{bulk})$$

Infinite ~~Finite~~-dimensional Hilbert space

Relative entropy conservation between bulk and boundary

JLMS formula

Entanglement wedge reconstruction

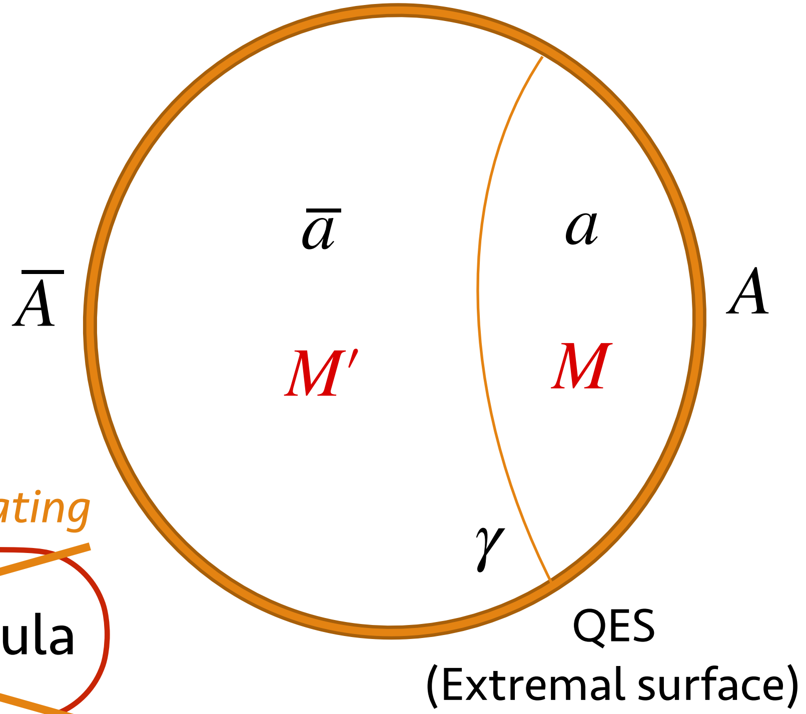
↔

Needs regulating

~~Ryu-Takayanagi Formula~~

Quantum extremal surface

[Swingle][Jafferis, Lewkowycz, Maldacena, Suh][Dong, Harlow, Wall]
[Harlow][MJK, Kolchmeyer'19][Gesteau, MJK'20'20][Faulkner]



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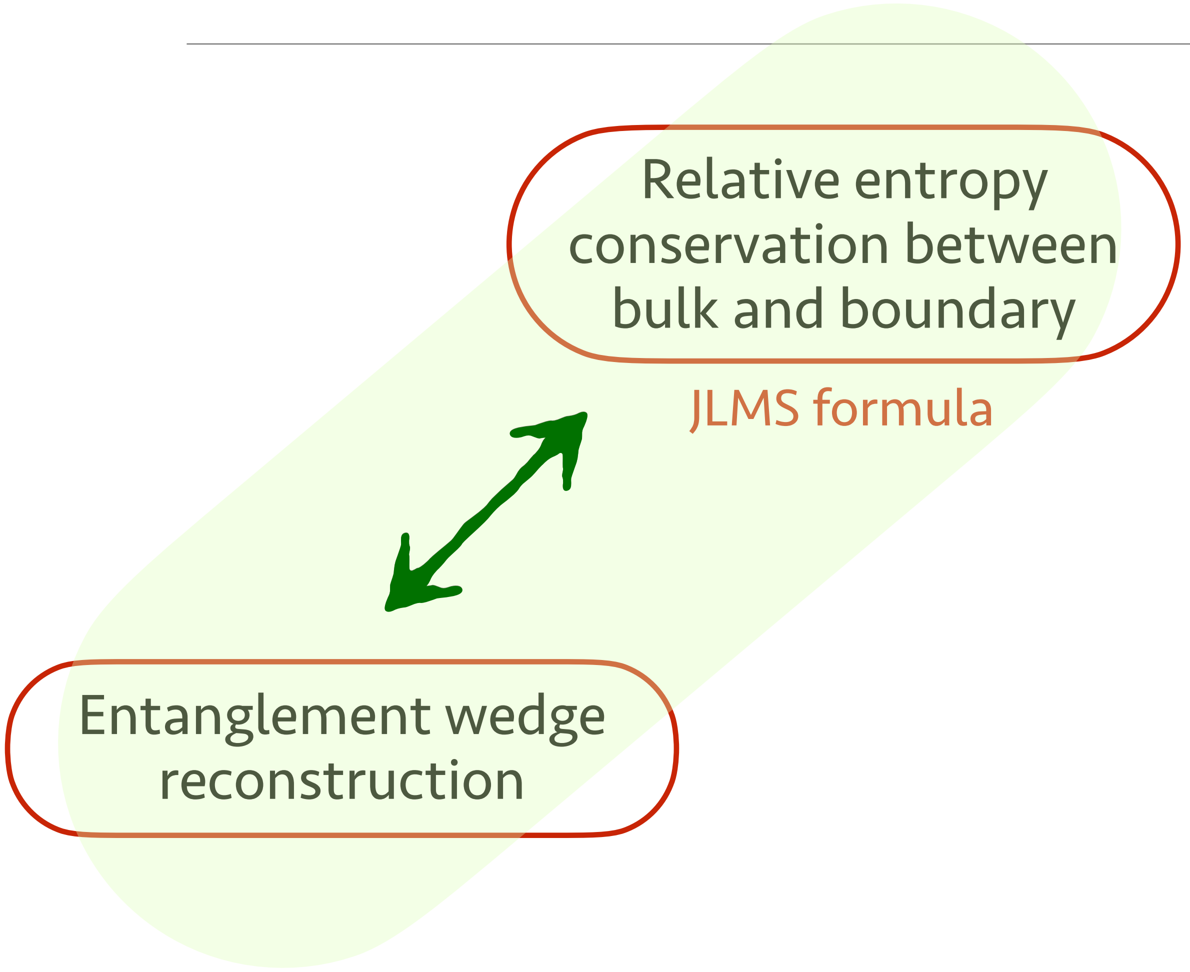
Infinite-dimensional Hilbert space



Can we have a relation between these two directly?

The exact relation: EWR=JLMS

Including a generic choice of infinite-dimensional Hilbert spaces

Entanglement wedge reconstruction = Complete recovery

[MJK, Kolchmeyer '18'19]
[Gesteau, MJK '20 '20 '21]
[Faulkner '20]

Relative entropy conservation between bulk and boundary

This is true to the order of G_N^0

[Kelly][Jafferis, Lewkowycz, Maldacena, Suh]

How about higher order of G_N?

Reconstruction wedge!

Finite: [Hayden, Penington]
Infinite: [Gesteau, MJK '21]

[MJK, Kolchmeyer '18]

Theorem. *Let $u : \mathcal{H}_{code} \to \mathcal{H}_{phys}$ be an isometry[2] between two Hilbert spaces. Let M_{code} and M_{phys} be von Neumann algebras on $\mathcal{H}_{code}$ and $\mathcal{H}_{phys}$ respectively. Let M'_{code} and M'_{phys} respectively be the commutants of M_{code} and M_{phys}. Suppose that the set of cyclic and separating vectors with respect to M_{code} is dense in $\mathcal{H}_{code}$. Also suppose that if $|\Psi\rangle \in \mathcal{H}_{code}$ is cyclic and separating with respect to M_{code}, then $u|\Psi\rangle$ is cyclic and separating with respect to M_{phys}. Then the following two statements are equivalent:*

1. **Bulk reconstruction**

$$\forall \mathcal{O} \in M_{code} \ \forall \mathcal{O}' \in M'_{code}, \quad \exists \tilde{\mathcal{O}} \in M_{phys} \ \exists \tilde{\mathcal{O}}' \in M'_{phys} \quad \text{such that}$$

$$\forall |\Theta\rangle \in \mathcal{H}_{code} \quad \begin{cases} u\mathcal{O}|\Theta\rangle = \tilde{\mathcal{O}}u|\Theta\rangle, & u\mathcal{O}'|\Theta\rangle = \tilde{\mathcal{O}}'u|\Theta\rangle, \\ u\mathcal{O}^\dagger|\Theta\rangle = \tilde{\mathcal{O}}^\dagger u|\Theta\rangle, & u\mathcal{O}'^\dagger|\Theta\rangle = \tilde{\mathcal{O}}'^\dagger u|\Theta\rangle. \end{cases}$$

2. **Boundary relative entropy equals bulk relative entropy**

For any $|\Psi\rangle, |\Phi\rangle \in \mathcal{H}_{code}$ with $|\Psi\rangle$ cyclic and separating with respect to M_{code},

$$\mathcal{S}_{\Psi|\Phi}(M_{code}) = \mathcal{S}_{u\Psi|u\Phi}(M_{phys}), \text{ and } \mathcal{S}_{\Psi|\Phi}(M'_{code}) = \mathcal{S}_{u\Psi|u\Phi}(M'_{phys}),$$

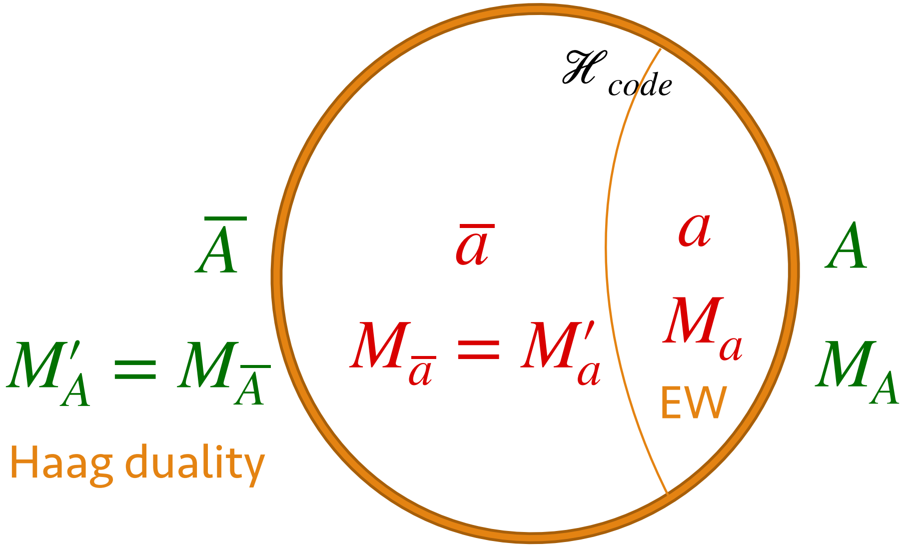
where $\mathcal{S}_{\Psi|\Phi}(M)$ is the relative entropy.

AdS/CFT interpretation

➡ $\mathscr{H}_{code} \implies$ a code subspace of the physical Hilbert space $\mathscr{H}_{phys}$ that consists of states with semi-classical bulk duals

AdS/CFT interpretation

➡ $\mathscr{H}_{code} \Longrightarrow$ a code subspace of the physical Hilbert space $\mathscr{H}_{phys}$ that consists of states with semi-classical bulk duals



$\mathscr{H}_{phys}$: the CFT Hilbert space

Boundary (physical) operators in region A: $\mathcal{O}$

in its complement $\bar{A}$: $\mathcal{O}'$

➡ Form vN algebras M_A and M'_A

Bulk (logical) operators in region a: $\tilde{\mathcal{O}}$

In its complement $\bar{a}$: $\tilde{\mathcal{O}}'$

➡ Form vN algebras M_a and M'_a

where a is the entanglement wedge of A

AdS/CFT interpretation

➡ **New insight**: this theorem provides **a necessary and sufficient criterion** for a subalgebra of bulk operators and its commutant to respectively be **reconstructed** in a subregion in the boundary and its complement.

➡ Ingredients of the theorem:

- An isometry $u : \mathscr{H}_{code} \to \mathscr{H}_{phys}$ (=the holographic map)
- Von Neumann algebras on $\mathscr{H}_{code}$ and $\mathscr{H}_{phys}$: $M_{code}, M'_{code}, M_{phys}, M'_{phys}$

➡ The only assumption required:

- If $|\Psi> \in \mathscr{H}_{code}$ is cyclic and separating with respect to M_{code}, then $u|\Psi>$ is cyclic and separating with respect to M_{phys}.

[MJK, Kolchmeyer '18]

Theorem. *Let $u : \mathcal{H}_{code} \to \mathcal{H}_{phys}$ be an isometry[2] between two Hilbert spaces. Let M_{code} and M_{phys} be von Neumann algebras on $\mathcal{H}_{code}$ and $\mathcal{H}_{phys}$ respectively. Let M'_{code} and M'_{phys} respectively be the commutants of M_{code} and M_{phys}. Suppose that the set of cyclic and separating vectors with respect to M_{code} is dense in $\mathcal{H}_{code}$. Also suppose that if $|\Psi\rangle \in \mathcal{H}_{code}$ is cyclic and separating with respect to M_{code}, then $u|\Psi\rangle$ is cyclic and separating with respect to M_{phys}. Then the following two statements are equivalent:*

There was one more actually. Can we relax that?

1. Bulk reconstruction

$$\forall \mathcal{O} \in M_{code}\ \forall \mathcal{O}' \in M'_{code}, \quad \exists \tilde{\mathcal{O}} \in M_{phys}\ \exists \tilde{\mathcal{O}}' \in M'_{phys} \quad \text{such that}$$

$$\forall |\Theta\rangle \in \mathcal{H}_{code} \quad \begin{cases} u\mathcal{O}|\Theta\rangle = \tilde{\mathcal{O}}u|\Theta\rangle, & u\mathcal{O}'|\Theta\rangle = \tilde{\mathcal{O}}'u|\Theta\rangle, \\ u\mathcal{O}^\dagger|\Theta\rangle = \tilde{\mathcal{O}}^\dagger u|\Theta\rangle, & u\mathcal{O}'^\dagger|\Theta\rangle = \tilde{\mathcal{O}}'^\dagger u|\Theta\rangle. \end{cases}$$

2. Boundary relative entropy equals bulk relative entropy

For any $|\Psi\rangle, |\Phi\rangle \in \mathcal{H}_{code}$ with $|\Psi\rangle$ cyclic and separating with respect to M_{code},

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where $\mathcal{S}_{\Psi|\Phi}(M)$ is the relative entropy.

[MJK, Kolchmeyer '18]

Theorem. *Let $u : \mathcal{H}_{code} \to \mathcal{H}_{phys}$ be an isometry between two Hilbert spaces. Let M_{code} and M_{phys} be von Neumann algebras on $\mathcal{H}_{code}$ and $\mathcal{H}_{phys}$ respectively. Let M'_{code} and M'_{phys} respectively be the commutants of M_{code} and M_{phys}.*
Suppose that

The only assumption is the existence of the state

- *There exists some state $|\Omega\rangle \in \mathcal{H}_{code}$ such that $u|\Omega\rangle \in \mathcal{H}_{phys}$ is cyclic and separating with respect to M_{phys}.*

- $\forall \mathcal{O} \in M_{code}\ \forall \mathcal{O}' \in M'_{code}$, $\exists \tilde{\mathcal{O}} \in M_{phys}\ \exists \tilde{\mathcal{O}}' \in M'_{phys}$ *such that*

Bulk reconstruction

$$\forall\, |\Theta\rangle \in \mathcal{H}_{code} \qquad \begin{cases} u\mathcal{O}\,|\Theta\rangle = \tilde{\mathcal{O}}u\,|\Theta\rangle\,, & u\mathcal{O}'\,|\Theta\rangle = \tilde{\mathcal{O}}'u\,|\Theta\rangle\,, \\ u\mathcal{O}^\dagger\,|\Theta\rangle = \tilde{\mathcal{O}}^\dagger u\,|\Theta\rangle\,, & u\mathcal{O}'^\dagger\,|\Theta\rangle = \tilde{\mathcal{O}}'^\dagger u\,|\Theta\rangle\,. \end{cases}$$

Then, for any $|\Psi\rangle, |\Phi\rangle \in \mathcal{H}_{code}$ with $|\Psi\rangle$ cyclic and separating with respect to M_{code},

- $u\,|\Psi\rangle$ *is cyclic and separating with respect to M_{phys} and M'_{phys},*
- $\mathcal{S}_{\Psi|\Phi}(M_{code}) = \mathcal{S}_{u\Psi|u\Phi}(M_{phys})$, $\quad \mathcal{S}_{\Psi|\Phi}(M'_{code}) = \mathcal{S}_{u\Psi|u\Phi}(M'_{phys})$,

Relative entropy conservation

where $\mathcal{S}_{\Psi|\Phi}(M)$ is the relative entropy.

The sketch of the proof

- We prove that for any $|\Psi\rangle \in \mathscr{H}_{code}$, which is cyclic and separating w.r.t. M_{code}, $u|\Psi\rangle$ is cyclic and separating w.r.t. M_{phys}. Then, we show that for any $\mathscr{P} \in M_{phys}$, $u^\dagger \mathscr{P} u \in M_{code}$.
- We relate $S^c_{\Psi|\Phi}$ and $S^p_{u\Psi|u\Phi}$ (which are relative Tomita operators defined w.r.t. M_{code} and M_{phys}) and derive $uS^c_{\Psi|\Phi} = S^p_{u\Psi|u\Phi}u$ for generically unbounded operators. We show that their domains are equal and $S^p_{u\Psi|u\Phi}$ restricted to the vector space $(\text{Im } u)^\perp$ has a range contained within $(\text{Im } u)^\perp$.
- We derive a relation for the relative modular operators associated with $S^c_{\Psi|\Phi}$ and $S^p_{u\Psi|u\Phi}$: $u\Delta^c_{\Psi|\Phi} = \Delta^p_{u\Psi|u\Phi}u$. This is related to the physical notion that bulk modular flow is dual to boundary modular flow. Likewise, we show that $\Delta^p_{u\Psi|u\Phi}$ restricted to the vector space $(\text{Im } u)^\perp$ has a range contained within $(\text{Im } u)^\perp$.
- We show that the spectral projections commute with the projector $uu^\dagger$. We derive that the spectral projections of $\Delta^c_{\Psi|\Phi}$ are given by $u^\dagger P^p_\Omega u$, where P^p_Ω denotes the spectral projections of $\Delta^p_{u\Psi|u\Phi}$.
- Any function of $\Delta^p_{u\Psi|u\Phi}$ or $\Delta^c_{\Psi|\Phi}$ can be constructed once the spectral projections are known. It follows that $\langle\Psi|\log \Delta^c_{\Psi|\Phi}|\Psi\rangle = \langle u\Psi|\log \Delta^p_{u\Psi|u\Phi}|u\Psi\rangle$, and thus the relative entropies are equal.

Limitations to the "exact" theorem

➡ The redundancy in the bulk-to-boundary encoding can only be consistent with the Reeh-Schlieder theorem on the boundary, if entanglement wedge reconstruction is approximate,

with an error that may be non-perturbative in G_N [Kelly]

➡ For state-independent recovery to be possible in an approximate setting, a local bulk operator has to be in the entanglement wedge of all pure and mixed states of the corresponding boundary region.

Reconstruction wedge [Hayden, Penington]

➡ The presence of nonperturbative errors in bulk reconstruction is crucial to the resolution of the black hole information paradox. [Penington]

(With non-perturbative gravity correction)

The "approximate" theorem

[Gesteau, MJK '21]

Approximate relative entropy conservation (=QES formula)

↕

Approximate recovery (=RW reconstruction)

$$|S_{\rho\circ(\mathscr{E}^c\otimes Id),\rho\circ(\mathscr{P}_{a'}\otimes Id)\circ(\mathscr{E}^c\otimes Id)}(M'_A\otimes\mathscr{B}(\mathscr{H}^*_{code}))-S_{\rho,\rho\circ(\mathscr{P}_{a'}\otimes Id)}(M_{EW(\rho\circ(\mathscr{P}_{a'}\otimes Id),\bar{A}\cup R)})$$
$$+S_{gen}(\rho,EW(\rho,\bar{A}\cup R))-S_{gen}(\rho,EW(\rho\circ(\mathscr{P}_{a'}\otimes Id),\bar{A}\cup R))|\leq\varepsilon$$

$$\hookrightarrow\ |S_{\rho\circ(\mathscr{E}^c\otimes Id),\rho\circ(\mathscr{P}_{a'}\otimes Id)\circ(\mathscr{E}^c\otimes Id)}(M'_A\otimes\mathscr{B}(\mathscr{H}^*_{code}))|\leq\varepsilon$$

↕

$$\|\mathscr{E}\circ\mathscr{R}-Id_{M_a}\|_{cb}\leq 2(2\varepsilon)^{\frac{1}{4}}$$

ε : non-perturbatively small in G_N ($\varepsilon\sim e^{-\kappa/G_N}$ $(\kappa>0)$)

Theorem 1.1. *Let $\mathcal{H}_{code}$ and $\mathcal{H}_{phys}$ be two Hilbert spaces, $V : \mathcal{H}_{code} \to \mathcal{H}_{phys}$ be an isometry, and $\mathcal{H}^*_{code}$ be any finite-dimensional Hilbert space of dimension smaller or equal to the one of $\mathcal{H}_{code}$. Let M_A be a von Neumann algebra on $\mathcal{H}_{phys}$. To each normal state ω in $\mathcal{B}(\mathcal{H}_{code} \otimes \mathcal{H}^*_{code})_*$, we associate two entanglement wedge von Neumann algebras $M_{EW(\omega,A)}$ and $M_{EW(\omega,\overline{A}\cup R)}$ of operators on $\mathcal{H}_{code} \otimes \mathcal{H}^*_{code}$, such that $M_{EW(\omega,A)} \subset \mathcal{B}(\mathcal{H}_{code}) \otimes Id$ and only depends on the restriction of ω to $\mathcal{B}(\mathcal{H}_{code})$, and $M_{EW(\omega,\overline{A}\cup R)} \subset M'_{EW(\omega,A)}$. Let*

$$M_a := \bigcap_{\omega} M_{EW(\omega,A)}$$

be the reconstruction wedge von Neumann algebra on $\mathcal{H}_{code}$, and suppose that $M_{a'}$, the relative commutant of M_a in $\mathcal{B}(\mathcal{H}_{code}) \otimes Id$, is a product of type I factors. Suppose that for all choices of $\mathcal{H}^*_{code}$ and all pairs of states ρ, ω in $\mathcal{B}(\mathcal{H}_{code} \otimes \mathcal{H}^*_{code})_*$ such that $S_{\rho,\omega}(M_{EW(\omega,\overline{A}\cup R)})$ is finite, we have the following modified-JLMS condition:

$$\begin{aligned}|S_{\rho\circ(\mathcal{E}^c\otimes Id),\omega\circ(\mathcal{E}^c\otimes Id)}(M'_A \otimes \mathcal{B}(\mathcal{H}^*_{code})) - S_{\rho,\omega}(M_{EW(\omega,\overline{A}\cup R)})\\ +S_{gen}(\rho, EW(\rho,\overline{A}\cup R)) - S_{gen}(\rho, EW(\omega,\overline{A}\cup R))| \ \leq \ \varepsilon,\end{aligned}$$

where $\mathcal{E}$ and $\mathcal{E}^c$ refer to the respective restrictions of $A \mapsto V^\dagger A V$ to M_A and M'_A, and the function $S_{gen}(\rho, EW(\omega,\overline{A}\cup R))$ depends only on the restrictions of ρ and ω to $M_{a'} \otimes \mathcal{B}(\mathcal{H}^*_{code})$. Then, there exists a quantum channel $\mathcal{R} : M_a \to M_A$ such that

$$\|\mathcal{E}\circ\mathcal{R} - Id_{M_a}\|_{cb} \leq 2(2\varepsilon)^{\frac{1}{4}}.$$

[Gesteau, MJK '21]

Privacy/correctability correspondence

Privacy/correctability Theorem by [Crann,Kribs,Levene,Todorov]

➢ A quantum channel $\mathcal{E} : M \to \mathcal{B}(\mathcal{H})$, a vN subalgebra $N \subset M$.

$N \subset \mathcal{B}(\mathcal{H})$ is $\varepsilon-$private for $\mathcal{E}$ ⟶ $N \subset \mathcal{B}(\mathcal{H})$ is $2\sqrt{\varepsilon}-$correctable for $\mathcal{E}$

$N \subset \mathcal{B}(\mathcal{H})$ is $8\sqrt{\varepsilon}-$private for $\mathcal{E}$ ⟵ $N \subset \mathcal{B}(\mathcal{H})$ is $\varepsilon-$correctable for $\mathcal{E}$

Dictionary

Holography	*Operator Algebras*
Boundary (physical) operators	*Von Neumann algebras* M_{phys}
Bulk (logical) operators	*Von Neumann algebras* M_{code}
Projections onto the bulk operators	*Conditional Expectation*
Exact bulk reconstruction	*Invariance under conditional expectation*
Relative entropy conservation between bulk and boundary (JLMS)	*Takesaki's theorem*
Petz map	*Generalized conditional expectation*
State-independent bound	*Completely bounded norm bound*
(Approximate) complementary recovery	*(ϵ-) Correctability/privaty duality*
Twirled Petz map	*Faulkner-Hollands map*

Conclusion

- ➢ **Operator algebras** naturally provide an excellent perspective to **holography**, and **holography** and **quantum error correcting structure** are intrinsically related.
- ➢ I have shown that entanglement wedge reconstruction (i.e. complete recovery) is equivalent to relative entropy conservation between bulk and boundary. [MJK, Kolchmeyer '18][Gesteau, MJK '20]
- ➢ Considering nonperturbative gravity corrections, now consider **approximate recovery**. Further, the privacy/correctability duality implies the state-independent reconstruction inside of the reconstruction wedge. [Gesteau, MJK '21]

Remark on spacetime changes

➢ We considered a single bulk of AdS/CFT, but we can naturally imagine a situation where there exist bulk topology changes.

➢ This naturally yields **baby universes**.

➢ We can analyze the Hilbert space of baby universes and study its physical implications using C*-algebras to describe them.

Theorem. *Let $\mathcal{A}_{QG}$ be the algebra of observables of quantum gravity, and suppose that $\mathcal{A}_{QG} = \mathcal{A}_{res} \otimes \mathcal{A}_{baby}$, where $\mathcal{A}_{baby}$ is a commutative C^*-algebra. Let ω be a state on $\mathcal{A}_{QG}$. The GNS representation of $\mathcal{A}_{baby}$ induced by ω has dimension 1 if and only if the restriction of ω to $\mathcal{A}_{baby}$ is pure.*

Theorem. *Let $\mathcal{A}_{QG}$ be the algebra of observables of bulk quantum gravity, and ω be a gravitational path integral state on $\mathcal{A}_{QG}$. Assume*

1. *for $\mathcal{A}_{baby}$ an Abelian C^*-algebra, $\mathcal{A}_{QG} = \mathcal{A}_{res} \otimes \mathcal{A}_{baby}$,*
2. *ω is a pure state on $\mathcal{A}_{QG}$,*
3. *ω can be factorized as $\omega = \omega_{res} \otimes \omega_{baby}$.*

Then, the GNS representation of $\mathcal{A}_{baby}$ is one-dimensional.

Many more exciting things!

- ➢ There are many exciting things that can be explicitly constructed from tensor network settings to give rise to toy models of quantum gravity.
  - Qutrit model [MJK, Kolchmeyer '19], Infinite-HaPPY code as a Trapeze model [Gesteau, MJK '20'20], Spacetime HaPPY code [MJK,Qasim (To appear)].
- ➢ Operator algebra has given new perspectives to global symmetry of the CFTs and their universal behaviors, and has shown to unveil more quantum error correcting structure of quantum gravity.

Thank you for listening!